

# GRAVITY'S SCALAR COUSIN

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## Abstract

The “dilaton”, the Goldstone boson of spontaneously broken conformal field theories (in flat spacetime), is argued to provide a surprisingly provocative scalar analog of gravity. Many precise parallels and contrasts are drawn. In particular, the Equivalence Principle, the Cosmological Constant Problem, and the tension between them is shown to be closely replicated. Also, there is a striking transition when mass is compressed within the (analog) Schwarzschild radius. The scalar analogy may provide a simpler context in which to think about some of the puzzles posed by real gravity.

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# 1 Introduction

A useful analogy must balance two requirements. It must obviously share some key features with the original system of interest. But it should also exhibit significant differences, which are either simplifying in nature, or at least place the shared features in an illuminating new context. For example, this is what we would hope for if ever we were to find extra-terrestrial life. When the system of interest is gravity, in either the classical or quantum sphere, a worthy analogy may seem very hard to come by. General Relativity is simply too special. Nevertheless, it is the purpose of this paper to describe just such an analogy, in the hope that it will contribute to understanding the deep mysteries that underlie the real thing.

The analog system is simply the low-energy effective description of a conformal field theory (CFT) in flat spacetime, whose conformal symmetry has somehow been spontaneously broken down to the usual Poincare symmetry. The low-energy “chiral Lagrangian” describes the general couplings of the “dilaton”, the scalar Goldstone boson of the symmetry breaking (our analog of the graviton), to other light remnants of the CFT (our analog of Standard Model matter). This paper will focus on pointing out the many surprising parallels between this scenario and real gravity. Important steps in this direction, including the development of the chiral Lagrangian, were taken long ago in Refs. [1] [2] [3] [4]. (Also see the discussions of scalar-to-gravity resemblance in Refs. [5] and [6].) Other ingredients appear in more recent literature. The synthesis presented here is hopefully novel. The remainder of this introduction will simply summarize the parallels and contrasts to be drawn in the course of the paper.

- The dilaton is a Goldstone boson and real gravitons have also been previously viewed as Goldstone bosons [2] [7]. This particular parallel receives no further discussion in the rest of the paper.

- In the chiral Lagrangian description (Sections 2, 3) there is an (emergent) fluctuating spacetime metric whose couplings to matter are constrained by general coordinate invariance [2] [3] [4] [8]. Consequently the Equivalence Principle is satisfied. There is a slight difference with real gravity (Sections 4, 5) in that there are small violations of the equivalence of gravitational and inertial masses due to the “gravitational” self-energies of massive bodies.

- Newton’s Law holds in the regime of non-relativistic matter, constrained by the Equivalence Principle (Section 4).

- The theory is fully relativistic and there are relativistic corrections to Newton’s Law but they differ in detail from those of General Relativity. There is dilaton radiation from accelerating matter but it differs from gravitational radiation in that it is scalar.

- When one tries to compress massive matter inside its Schwarzschild radius, something interesting happens (Section 5). In real gravity a black hole appears. In the dilaton theory the

chiral Lagrangian description itself breaks down and the full CFT dynamics becomes important. It is possible that a black-hole-like phenomenon still takes place. This is not proven but some evidence presented (Section 15).

The holographic principle of real gravity [9] states that there are far fewer degrees of freedom than naively appear in in our usual way of thinking about effective field theory. In the dilaton theory there is a sign that the naive number of degrees of freedom in the regime of validity of the effective field theory is a gross overestimate, but this requires further study (Section 5).

- Light does not bend (classically) in a dilaton field (Section 6), unlike the classic effect in General Relativity. Therefore, there are no black holes in the conventional sense. Quantum effects can cause light to bend (Section 7.1).

- The chiral Lagrangian description is a non-renormalizable effective quantum field theory which breaks down above the analog Planck scale just as effective General Relativity does (Section 7). The UV completion of the effective theory is a fundamental CFT with a moduli space (Section 11). For gravity the known UV completions are string theories. In both cases, supersymmetry appears in the known constructions. Remarkably, in some cases the two types of UV completions are related (Section 16) via the AdS/CFT correspondence [10] (although the gravity is then higher dimensional).

- There is an analog of the cosmological constant which obstructs Poincare invariant solutions, but does give rise to solutions which are seen by matter to be de Sitter or anti-de-Sitter spacetimes (Section 8). Observers in the de Sitter solution will see Hawking radiation due to the cosmological horizon. Unlike real gravity, positive vacuum energy leads to the  $AdS$  solution and negative vacuum energy leads to the  $dS$  solution.

- General homogeneous and isotropic cosmologies can also be studied (Section 9) in the dilaton chiral Lagrangian description. They are quite distinct from standard general relativistic cosmologies for simple reasons.

- There is a version of the Cosmological Constant Problem (Section 10), the matter contributions to the analog cosmological constant being *identical* to the form of such contributions to the real cosmological constant (in the absence of subleading quantum dilaton and quantum gravity corrections).

- The analog Planck length acts as a minimal length scale (Section 12), certainly in the dilaton effective theory, and even in its UV completion.

- Recently, effective field theory sense has been made of the notion of graviton mass [11] at the cost of exact general covariance, although the nature of any UV completion is unknown. In contrast there is a natural way in which a dilaton mass could arise (Section 13), but again at the cost of exact general covariance in the couplings to matter.

- The supersymmetric version of the chiral Lagrangian description is straightforward to

write down (Section 14) and is the analog of supergravity coupled to supersymmetric matter. Supersymmetry can technically protect the analog cosmological constant until supersymmetry is broken. There are interesting ways to do this.

- One can prove in the analog system that supersymmetry breaking implies the breaking of Poincare invariance [12] (Section 14), a property conjectured to be true in string theory [13].

- The supergravity mechanism of “anomaly-mediation” [14] has rather close analogs in both the supersymmetric (Section 14) and non-supersymmetric (Section 7) effective descriptions of the dilaton coupled to quantum matter.

- Just as the Randall-Sundrum II model [15] demonstrates the mechanism of localized 4D gravity in higher dimensions, there is a simple RS construction of a localized 4D dilaton (coupled to localized matter) in higher dimensions (Section 15). In both cases the AdS/CFT correspondence illuminates the localization mechanism (Section 16).

- While attempts to define Euclidean quantum effective gravity are troubled by an IR problem, namely the unboundedness of the Euclidean Einstein action [16], the Euclideanized theory based on the dilaton chiral Lagrangian is well defined in the IR (Section 17). Of course both real and analog gravity suffer in the UV from non-renormalizability.

Effective field theories of the dilaton coupled to matter are sufficiently rich that they can mimic our real universe in its everyday running. There is no difficulty at the level of the chiral Lagrangian in describing fields and interactions that support stars, solar systems and life. Indeed, at the dawn of relativity, scalar theories were seriously considered as realistic candidates for gravitational dynamics [17].

## 2 Dilaton Chiral Lagrangian

Here, and in the next section, we will develop the chiral Lagrangian corresponding to the spontaneous breakdown of conformal symmetry to standard Poincare symmetry (in Minkowski spacetime). For the most part, this is a review of Refs. [2] [3] [4], but given in the most suggestive language for our purpose. (In general, there are many possible, physically equivalent, formalisms for chiral Lagrangians, connected by field redefinitions [18].)

The fifteen parameter conformal group is the subgroup of general coordinate transformations that result in an overall coordinate-dependent rescaling of the Minkowski metric:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu =_{x'(x)} f(x') \eta_{\mu\nu} dx'^\mu dx'^\nu. \quad (2.1)$$

It consists of the Poincare transformations, rigid scale transformations and the special conformal

transformations,

$$x'^{\mu} \equiv \frac{x^{\mu} + x^2 b^{\mu}}{1 + 2b \cdot x + b^2 x^2}. \quad (2.2)$$

It is generally believed that any unitary theory possessing Poincare and scale symmetry will automatically be invariant under the special conformal transformations as well. For a discussion see Ref. [19]. Relatedly, a single scalar Goldstone field for broken scale invariance, the “dilaton”, is all that is needed in order to non-linearly realize conformal invariance in the chiral lagrangian. The extra four Goldstone bosons corresponding to broken special conformal generators can be taken as the derivatives of the dilaton rather than independent fields [2]. The breakdown of the one-to-one correspondence between Goldstone fields and broken generators can occur when these generators are constructed from fewer conserved currents, as can happen with spacetime symmetries [20].

We will employ a dimensionless interpolating field for the dilaton,

$$\phi(x) \equiv e^{\pi(x)/M}, \quad (2.3)$$

where  $M$  is the order parameter scale of conformal invariance breaking, and  $\phi(x)M$  describes Goldstone fluctuations about this vacuum, with canonical field  $\pi(x)$ . We clearly are expanding about the VEV  $\langle \phi \rangle = 1$ , any other constant choice being absorbable into  $M$ .  $\phi$  (or  $\pi$ ) will be our analog scalar graviton and  $M$  will play an analogous role to the Planck scale in real gravity. The quantitative value of the scale  $M$  is arbitrary by the fundamental scale invariance of the dynamics but for the sake of our analogy we will take it to be the same as the real Planck scale.

The non-linear Goldstone transformation law under conformal symmetry is conveniently taken to be

$$\phi'(x') = \sqrt{f(x')} \phi(x), \quad (2.4)$$

where  $f$  is given in Eq. (2.1). Thus for rigid scale transformations for example,  $x' = \lambda x$ ,

$$\pi'(\lambda x) = \pi(x) - M \ln(\lambda). \quad (2.5)$$

This is similar to the usual shift transformations of spontaneously broken internal symmetries, except that the shift in the field is accompanied by a change of the coordinate on the left-hand side. This last point is important when we write the most general chiral lagrangian for the dilaton because, unlike Goldstone bosons of internal symmetries, we are now able to write a (unique form of) dilaton potential,

$$S_{dilaton} = \int d^4x \left\{ \frac{M^2}{2} (\partial_{\mu} \phi)^2 - \Lambda \phi^4 + \text{higher derivatives} \right\}. \quad (2.6)$$

The conformal invariance is straightforward to check.

The appearance of the non-derivative coupling is unusual. Normally when we contemplate a symmetry of the dynamics, it seems a robust possibility that it is realized in a spontaneously broken phase. Then we can write a chiral lagrangian for the requisite Goldstone field and its derivative couplings ensure that the Goldstone field manifold describes degenerate vacua. In particular any choice of Goldstone VEV is a suitable vacuum choice. In the present case with the  $\phi^4$  coupling, this is false, and in fact it results in a runaway behavior,  $\phi \rightarrow \infty, 0$ , where the conformal invariance is either broken at infinitely high energy or not broken at all. In either case we are driven out of the useful regime for a chiral lagrangian. However, it may be that  $\Lambda = 0$  ( $\Lambda \ll M^4$ ), so that our vacuum choice,  $\langle \phi \rangle = 1$ , is (approximately) stable. Very naively, we would have expected  $\Lambda \sim \mathcal{O}(M^4)$  in the chiral lagrangian, which shows us that the broken phase of conformal invariance requires special circumstances, not as robust as with other symmetries. We shall nevertheless proceed by assuming that  $\Lambda \ll M^4$ .

A first connection with standard General Relativity arises by constructing a fluctuating “auxiliary” (as opposed to independent) metric out of the dilaton,

$$g_{\mu\nu}(x) \equiv \phi^2(x)\eta_{\mu\nu}, \quad (2.7)$$

and noting that the standard metric transformation law and the dilaton transformation precisely agree under conformal coordinate transformations. In terms of the auxiliary metric we can re-write the chiral lagrangian,

$$S_{dilaton} = \int d^4x \sqrt{-g} \left\{ -\frac{M^2}{12} R - \Lambda + \text{higher derivatives} \right\}. \quad (2.8)$$

Of course it is understood that in varying this action  $\phi(x)$  is the independent variable, not  $g_{\mu\nu}$ . In fact, the curvature (kinetic) term above has the opposite sign from the standard Einstein action. The latter sign in General Relativity gives positive energies to physical gravitational fluctuations, while fluctuations of the form  $g_{\mu\nu}(x) \equiv \phi^2(x)\eta_{\mu\nu}$  appear to have negative-definite gradient energies, but fortunately these fluctuations are pure gauge. However in the dilaton theory, the dilaton is physical and there are no other fluctuations, and bounded energy is achieved by a sign flip.

Despite the general covariance of the action, Eq. (2.8), general covariance is not respected by the condition, Eq. (2.7). However we can reformulate the chiral lagrangian in a completely generally covariant fashion by taking  $g_{\mu\nu}(x)$  to be the fundamental field, and imposing the generally covariant subsidiary condition that the Weyl tensor vanishes,

$$R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \frac{R}{6}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}) = 0. \quad (2.9)$$

Metrics satisfying this condition are precisely those which can be expressed in the form, Eq. (2.7), in some coordinate system [21]. Thus if we extremize the action for metrics satisfying this

condition, we will recover the classical content of the dilaton theory. At the level of quantum effective field theory we could impose the vanishing of the Weyl tensor in a path integral over metrics, which would be equivalent to integrating over  $\phi$  along with some coordinate gauge redundancy, which can be gauge fixed in the usual way. For the rest of this paper however, it is sufficient to think of  $\phi$  as the fundamental field and  $g_{\mu\nu}$  as an auxiliary construct, useful in adding other light matter ( $\ll M$ ) of the broken conformal theory to the chiral lagrangian description, as we will now see.

### 3 Matter and the Equivalence Principle

Once conformal invariance is spontaneously broken, one expects  $M$  to set the mass gap for generic states made from the conformal theory. However there may be states which are much lighter than  $M$ , protected by infrared symmetries or by coincidence. We must therefore include this light matter sector in our chiral lagrangian description below  $M$ , coupling it to the dilaton in the most general conformally invariant way. As usual, weakly coupled matter can only be scalars  $\chi$ , fermions  $\psi$ , and vector fields  $A_\mu$ . This matter will be our analog of Standard Model matter in the real world. We can write the general<sup>1</sup> conformally invariant low-energy effective action in the (notationally-condensed) form,

$$\begin{aligned}
S_{eff} = & \int d^4x \sqrt{-g} \left\{ -\frac{M^2}{12} R + k_1(\chi, \psi) g^{\mu\nu} D_\mu \chi^* D_\nu \chi + k_2(\chi, \psi) \bar{\psi} i e^\mu_a D_\mu \gamma^a \psi - V(\chi, \psi) \right. \\
& - k_3(\chi, \psi) g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + i k_4(\chi, \psi) \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} F_{\mu\nu} F_{\alpha\beta} - k_5(\chi, \psi) \bar{\psi} \sigma^{ab} \psi F_{\mu\nu} e^\mu_a e^\nu_b \\
& \left. + k_6(\chi, \psi) e^\mu_a D_\mu \chi \bar{\psi} \gamma^a \psi + \text{higher derivatives} \right\}.
\end{aligned} \tag{3.1}$$

The explicitly written terms are up to two-derivative order (one-derivative order when there are fermions present). The covariant derivative for fermions hides a spin connection based on the auxiliary vierbein,  $e_\mu^a \equiv \phi(x) \delta_\mu^a$ , but the scalar (gauge-covariant) derivatives and gauge field strengths are independent of the metric (dilaton).

This action is obviously invariant under conformal transformations when we take the matter fields to transform in the standard way under *general* coordinate transformations, treating conformal transformations as a subgroup. In fact the action is invariant under general coordinate transformations. However, one might think that there are other conformally invariant terms possible which cannot be written as general coordinate invariants. But this is not the case to

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<sup>1</sup>Without loss of generality, we have written the chiral lagrangian in “Einstein frame” because the Weyl transformations of the metric needed to go to this frame obviously correspond to well-defined transformations on the dilaton by Eq. (2.7).

two-derivative order [4]. One, more or less brute force, way to see this is to systematically list all terms of this order subject only to rigid scale and Poincare invariance. All terms which are independent of those in Eq. (3.1) then contain derivatives of  $\phi$ , are straightforwardly seen to be non-invariant under special conformal transformations and are therefore excluded. Thus full general coordinate invariance is an accidental (gauge) symmetry of matter couplings at two-derivative order. In fact I suspect this remains true to all orders, but there is as yet no general proof of this. For our purposes it will be sufficient to work to two-derivative order and treat higher orders as “beyond experimental precision”.

Accidental general coordinate invariance for the matter couplings to the dilaton clearly translates into an accidental Equivalence Principle for light matter emerging from the broken conformal theory. That is, light matter sees the dilaton only via the generally covariant couplings to the auxiliary metric  $g_{\mu\nu}$ . For a fixed dilaton field, non-linearly realized conformal invariance forces the matter fields to propagate and interact “as if” they were in a curved space with the auxiliary metric.

## 4 Newton’s Law

Let us consider first a simple example with a single scalar species of matter,  $\chi$ . Eq. (3.1) clearly allows matter to have mass (since conformal invariance is spontaneously broken). Suppose there are several non-relativistic  $\chi$  particles in some reference frame, widely-separated so that we can neglect  $\chi$  self-interactions. We also take  $\Lambda$  (now thought of as the VEV of  $V(\chi)$ ) to be negligibly small. Then Eq. (3.1) can be written as

$$\mathcal{L}_{eff} = \frac{\phi^2}{2}(\partial\chi)^2 - \frac{m^2}{2}\phi^4\chi^2 + \frac{M^2}{2}(\partial\phi)^2. \quad (4.1)$$

Doing the field redefinition  $\chi\phi \rightarrow \chi$  and some integration by parts,

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\chi)^2 - \frac{m^2\phi^2}{2}\chi^2 + \chi^2\frac{\partial^2\phi}{2\phi} + \frac{M^2}{2}(\partial\phi)^2. \quad (4.2)$$

This contains trilinear vertices  $-m^2\pi(x)\chi^2/M + \chi^2\partial^2\pi/2M$  from which we can construct one-dilaton exchange diagrams between a pair of  $\chi$  particles. Clearly such exchanges will be ultra-local if we use the second vertex, so we use just the first. After amputating external  $\chi$  legs we get  $m^2/(M^2\bar{q}^2)$ , corresponding to a non-relativistic Newtonian potential in position space,  $m^2/(M^2r)$ , and confirming our earlier claim that  $M$  is the analog “Planck scale”.

This result is much more general. There may be several types of non-relativistic particles,  $\Psi$ , of mass  $m$  which can carry spin and may even be composites of the fundamental fields



$\chi, \psi, A_\mu$ . We can write a heavy particle effective theory [22], constrained only by the fact that all matter sees the dilaton via generally covariant couplings to  $g_{\mu\nu} \equiv \phi^2 \eta_{\mu\nu}$ ,

$$\mathcal{L}_{eff} = \sqrt{-g} \left\{ \frac{g^{\mu\nu}}{2m} (\partial_\mu - imv_\mu) \Psi_v^\dagger (\partial_\mu + imv_\mu) \Psi_v - \frac{m}{2} \Psi_v^\dagger \Psi_v + \text{less relevant} \right\}, \quad (4.3)$$

where  $v_\mu$  is a four-velocity defining a frame in which the  $\Psi$  are non-relativistic. Such contributions for different species of  $\Psi$  are implicitly taken to be summed here. Spin degrees of freedom decouple at leading order. A derivation of such effective lagrangians in the general relativistic context is given in Ref. [23], based on earlier discussions in [24] [25]. After field redefining  $\Psi_v \rightarrow \Psi_v/\phi$ , the canonical (heavy particle) fields have a leading trilinear vertex,  $m\pi(x)|\Psi_v|^2/M$ , which results again in the general form of Newton's Law,  $mm'/(M^2 r)$ .

Of course, the Newtonian approximation displays the non-relativistic face of the Equivalence Principle. Even though relativistic effects distinguish the dilaton theory from real gravity, the Equivalence Principle remains true in the relativistic regime in both theories by (accidental) general covariance.

There is one important difference between scalar and real gravity when we cannot neglect the gravitational self-energy of the non-relativistic “particles”, for example when these “particles” are whole planets or stars. In real gravity our effective Lagrangian continues to hold because full general covariance is exact. However, for scalar gravity the derivation of the effective Lagrangian only holds when the auxiliary metric is treated as a background field. (Of course once derived we can use it to derive Newton's Law by integrating out  $\phi$ .) Therefore it does not hold when gravitational self-energy is significant. Still, these self-energies are typically small and many tests of the Equivalence Principle (say rates of fall of modest masses in the Earth's gravitational field) are experimentally insensitive to this difference, so the analogy is good. We will explicitly see in the next section that there is a (small) inequivalence of gravitational and inertial masses when gravitational self-energy is taken into account, in contrast to real General Relativity.

## 5 At the Schwarzschild Radius

For simplicity, let us determine the spherically symmetric dilaton field about a point mass. While quantum matter fields ultimately do describe point particles, the most direct approach to the classical regime is to employ the classical point-particle action functional of the particle

worldline  $x^\mu(\tau)$ ,

$$\begin{aligned} S_{particle} &= -m \int d\tau \sqrt{g_{\mu\nu}(x(\tau)) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} \\ &= -m \int d\tau \phi(x(\tau)) \sqrt{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}. \end{aligned} \quad (5.1)$$

In the absence of  $\Lambda$  and in the static limit for the particle-dilaton system,  $t(\tau) = \tau$ ,  $\vec{x}(\tau) = \vec{0}$ ,  $\phi = \phi(\vec{x})$ , the dilaton equation of motion reads,

$$\nabla^2 \phi = \frac{m}{M^2} \delta^3(\vec{x}). \quad (5.2)$$

The spherically symmetric solution in polar coordinates is then

$$\phi(r) = 1 - \frac{m}{4\pi M^2 r}, \quad (5.3)$$

where the behavior at infinity is chosen to match onto the vacuum  $\phi = 1$ . Note that this is an exact solution of the equations of motion to leading order in the derivative expansion.

Now let us try to compute the mass of the particle including its dilaton field and see if it matches the “gravitational mass” setting the coefficient of the  $1/r$  fall off, which is clearly just  $m$ . The Hamiltonian of our theory is given (after implementing the static ansatz) by

$$\begin{aligned} H &= \int d^3\vec{x} \left\{ \frac{M^2}{2} (\nabla\phi)^2 + m\phi\delta^3(\vec{x}) \right\} \\ &= \int d^3\vec{x} \left\{ -\frac{M^2}{2} \phi \nabla^2 \phi + m\phi\delta^3(\vec{x}) \right\} + \frac{M^2}{2} \int^\infty d^2\vec{S} \cdot \phi \nabla \phi, \end{aligned} \quad (5.4)$$

where we have performed an integration by parts in the second equality and the last term is the surface term at infinity. Now let us plug in our dilaton solution to get the rest-energy or mass, using the equation of motion to simplify the computation of the volume term,

$$E = \frac{m}{2} (1 + \phi(0)). \quad (5.5)$$

This is ill-defined because  $\phi$  diverges at the origin. If we consider our calculation to be only an approximation to a finite-sized but compact mass, then the divergence is cut off by some length scale,  $r_m$ ,

$$E = m \left( 1 - \frac{m}{8\pi M^2 r_m} \right). \quad (5.6)$$

This exhibits a non-vanishing gravitational correction relative to the gravitational mass,  $m$ . By contrast in ordinary General Relativity the gravitational mass setting the  $1/r$  fall-off is exactly the same as rest-energy of the system.

There is a second type of singularity in our solution which is more alarming. Our dilaton solution passes through zero at a finite distance from the mass, of order the usual Schwarzschild radius,

$$r_S = \frac{m}{4\pi M^2}. \quad (5.7)$$

Since the local scale of spontaneous breaking of conformal invariance is set by  $\phi(x)M$ , whose non-vanishing justifies our chiral Lagrangian description, we really cannot trust our solution near or inside  $r_S$ . This is in contrast to General Relativity where curvatures are low at the Schwarzschild radius and the Schwarzschild solution can still be trusted. However, the two types of gravity are still somewhat analogous in that something very interesting happens at the Schwarzschild radius in each case when one compresses mass within or near this radius.

Of course the simple way to avoid this singularity is to consider a mass of finite size larger than  $r_S$ , so that the interior solution is modified and all singularities smoothed out, as for example would be the case for any conventional star or planet. I do not know how to derive any exact solutions for finite size objects with some reasonable equation of state, although one can work perturbatively. But one might also wonder what happens if matter collapses inside  $r_S$ . There appears to be no robust answer without reference to the details of the conformal theory in its symmetric phase, which is being restored as  $\phi \rightarrow 0$  near  $r_S$ . One expects the fundamental conformal theory beyond the chiral Lagrangian description to become important near  $r_S$  and to resolve all singularities. In Section 15 we will see some hints that the nature of this resolution is to replace the singularities by something like a black hole, but not black holes derivable purely within the chiral Lagrangian (which obviously do not exist).

Since the dilatonic effective theory does not possess black holes, the usual arguments in favor of the Holographic Principle [9] do not apply. And yet, as we have seen, the effective field theory breaks down if one tries to pack mass inside its “Schwarzschild radius”,  $r_S$ . That is, most of the naive states of the effective field theory are in fact not sensibly described by it and the full CFT must take over. This reduction of the degrees of freedom within effective theory control would be interesting to study more precisely.

## 6 Light Unbent

Our dilaton theory is fundamentally relativistic and has the same non-relativistic limit as ordinary gravity. However, of course, the relativistic details differ, unlike real gravity there is only scalar “gravitational” radiation here. Perhaps more significantly, light does not bend in a dilaton field, where “light” can refer to any free (at low energies) massless vector field surviving conformal symmetry breaking in the CFT. In fact there is no interaction with the dilaton at all.

This is easy to see, and well known as the Weyl invariance of the minimally-coupled Maxwell action:

$$\begin{aligned}\mathcal{L}_{light} &= -\sqrt{-g}\frac{g^{\mu\alpha}g^{\nu\beta}}{4}F_{\mu\nu}F_{\alpha\beta} \\ &= -\frac{\eta^{\mu\alpha}\eta^{\nu\beta}}{4}F_{\mu\nu}F_{\alpha\beta}.\end{aligned}\tag{6.1}$$

Of course, the dilaton will couple to light in higher dimension effective operators such as

$$\mathcal{L}_{higher-dim.} \propto \sqrt{-g}Rg^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta},\tag{6.2}$$

but the effect rapidly becomes negligible for dilaton fields softer than the scale suppressing the higher dimension operator. However, there are circumstances where such effects could be magnified. For example in the dilaton field due to a star not much larger than  $r_S$ ,  $\phi$  becomes small and the higher derivative operator becomes important because it scales like  $1/\phi^2$ . The qualitative difference with General Relativity is that this bending cannot be predicted quantitatively within the effective theory since it arises from non-minimal higher dimension operators. Again, in Section 15 we will see that with some more information about the fundamental conformal dynamics comes a greater level of predictivity.

## 7 Quantum Effective Field Theory

The dilaton couplings are certainly non-renormalizable, but just like General Relativity coupled to matter, they can be treated by the standard methods of quantum effective field theory in the sub-Planckian regime [26]. The consistency of the (non-linearly realized) conformal invariance of the effective theory in the quantum regime is easy to prove. Using our Goldstone field, the dilaton, dimensional regularization can be made fully covariant. The simplest way to see this is to work in the auxiliary metric language and dimensionally regulate in the manner familiar in General Relativity,

$$\begin{aligned}\mathcal{L}_{(4+\epsilon)D} &= \mu^\epsilon\sqrt{-g}\{-\frac{M^2}{12}R + k_1(\chi, \psi)g^{\mu\nu}D_\mu\chi^*D_\nu\chi + k_2(\chi, \psi)\bar{\psi}ie^\mu{}_aD_\mu\gamma^a\psi - V(\chi, \psi) \\ &\quad - k_3(\chi, \psi)g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - k_5(\chi, \psi)\bar{\psi}\sigma^{ab}\psi F_{\mu\nu}e^\mu{}_ae^\nu{}_b \\ &\quad + k_6(\chi, \psi)e^\mu{}_aD_\mu\chi\bar{\psi}\gamma^a\psi + \text{higher derivatives}\},\end{aligned}\tag{7.1}$$

where  $g_{\mu\nu} \equiv \phi^2(x)\eta_{\mu\nu}$  now is a  $(4+\epsilon) \times (4+\epsilon)$  matrix,  $\mu$  is the RG scale, and we have dropped the  $k_4$  term of Eq. (3.1) in order to avoid the usual issue to do with dimensionally continuing the  $\epsilon$ -symbol. The central impact of this is that now  $\sqrt{-g} = \phi^{4+\epsilon}$ , the extra  $\phi^\epsilon$  multiples

$\mu^\epsilon$ , effectively turning the latter into a spacetime dependent RG scale! Thus the naive lack of conformal invariance in general matter couplings arising from the usual scale anomaly, and tracked by  $\mu$ -dependence, is precisely cancelled by the accompanying  $\phi$ -dependence. A related discussion in a different context appears in Ref. [27]. The way in which all this works out after renormalization is simply illustrated by considering a matter sector consisting of QED, to which we now turn.

## 7.1 Anomaly-mediated bending of light

We will consider here quantum matter in the form of QED, coupled to a soft dilaton background. We can neglect the quantum dilaton backreaction if we consider QED processes far below  $M$ . For a constant  $\phi$  background the (renormalized) vacuum polarization is easily seen to be the same as in pure QED but with the rescalings  $m_{electron} \rightarrow m_{electron}\phi$ ,  $\mu \rightarrow \mu\phi$ ,

$$\Pi_{\mu\nu}(q) = \frac{e^2(\mu)}{2\pi^2}(\eta_{\mu\nu}q^2 - q_\mu q_\nu) \int_0^1 dx x(1-x) \ln\left(\frac{m^2\phi^2 - q^2x(1-x)}{\mu^2\phi^2}\right). \quad (7.2)$$

First consider the case  $m_{electron} = 0$ . Then in position space the renormalized effective action (following from the above vacuum polarization) is given by

$$\Gamma = \int d^4x \frac{e^2(\mu)}{6\pi^2} \ln(\mu\phi) F_{\mu\nu}(x) F^{\mu\nu}(x) + \mu\phi - \text{independent}. \quad (7.3)$$

(Note, the  $\mu\phi$ -independent terms contain finite non-local pieces as well as the classical action.) This remains true even when  $\phi(x)$  is slowly varying in spacetime,

$$\Gamma = \frac{e^2(\mu)}{6\pi^2} \ln(\mu\phi(x)) F_{\mu\nu}(x) F^{\mu\nu}(x) + \mu\phi - \text{independent}, \quad (7.4)$$

the locality of the  $\phi$ -dependent terms guaranteed by the locality of the ultraviolet divergences that necessitate the accompanying  $\mu$ -dependence. Note that although the effective action is non-analytic in  $\phi$ , as long as we are expanding around a vacuum  $\langle\phi\rangle \neq 0$ , we can expand in a power series in the canonical  $\pi(x)$  field, that is effective vertices involving  $\pi$ . The quantum effects of massless charges cause light to bend in a dilaton field!

We see here that the interacting photon does couple to the dilaton in order to cancel the conformal anomaly in pure QED, and we can therefore use the RG functions which describe this anomaly to determine the dilaton coupling. We therefore say that this sensitivity to the dilaton is “anomaly-mediated”. There is a closely related phenomenon in standard supergravity, “anomaly-mediated supersymmetry breaking” [14], where instead of tracking matter sensitivity to the  $x$ -dependence of the dilaton background we track matter sensitivity to the supergravity

background dependence on superspace Grassman coordinates. The role of the dilaton is played by the auxiliary chiral supermultiplet of supergravity known as the “compensator” [28].

Now consider the case  $m_{electron} \neq 0$ . Then for soft light the  $\phi$ -dependence cancels out of the vacuum polarization and the photon is decoupled from the dilaton even at the one-loop level. The higher dimension operators such as that in Section 6 can be induced also by loops upon integrating out the electron. We have neglected such effects in computing the polarization by treating  $\phi$  as “nearly” constant.

## 7.2 Quantum dilaton effects

There is no obstacle to including dilaton internal lines in Feynman diagrams. Dimensional regularization continues to provide a conformally invariant regularization when implemented as described above. Just as for standard quantum gravity corrections to Feynman diagrams, dilaton exchanges are Planck-suppressed and therefore irrelevant in the far infrared. It is only when considering such effects that the non-renormalizability of the effective theory becomes an essential complication and the theory must be treated by the standard methods of non-renormalizable effective field theory. But there are no extra subtleties compared with the treatment of quantum gravity [26] in this regard, in fact there are fewer.

## 8 Symmetric Spacetimes and Cosmological Horizons

In previous sections we have mostly neglected the non-derivative dilaton coupling,  $\Lambda$ . Let us now consider simple solutions in its presence. In the absence of matter (or after integrating out matter effects), the dilaton equations of motion are

$$\partial^2 \phi = -\frac{4\Lambda}{M^2} \phi^3. \quad (8.1)$$

Clearly if  $\Lambda \neq 0$  there are no Poincare invariant solutions with spontaneously broken conformal invariance,  $\phi \neq 0$ . This is analogous to the absence of Poincare invariant solutions in ordinary gravity when the cosmological constant is non-zero. However, there are simple solutions without Poincare invariance,

$$\phi = \frac{M}{\sqrt{-2\Lambda t}}, \quad \Lambda < 0 \quad (8.2)$$

$$\phi = \frac{M}{\sqrt{2\Lambda z}}, \quad \Lambda > 0 \quad (8.3)$$

where  $t$  (or  $-t$ ) is Minkowski time, and  $z$  is a Minkowski spatial coordinate. When matter is part of the effective theory it only sees the auxiliary metrics,

$$g_{\mu\nu} = -\frac{M^2}{2\Lambda t^2}\eta_{\mu\nu}, \quad \Lambda < 0 \quad (8.4)$$

$$g_{\mu\nu} = \frac{M^2}{2\Lambda z^2}\eta_{\mu\nu}, \quad \Lambda > 0, \quad (8.5)$$

which are (patches of) the maximally symmetric spacetimes,  $dS_4$  and  $AdS_4$  respectively.

Clearly  $\Lambda$  behaves quite analogously to the cosmological constant in ordinary gravity, and from now on we will refer to it as such. One difference to note is that the  $dS_4$  is associated here with  $\Lambda < 0$  and  $AdS_4$  with  $\Lambda > 0$ , the opposite of the familiar correlation in gravity. The reason is simply traced to the fact that when written in terms of the auxiliary metric the dilaton kinetic term has the opposite sign from the usual Einstein action for the reasons discussed at the end of Section 2.

Observers made out of light matter in our  $dS_4$  auxiliary spacetime will see the usual cosmological horizon and quantum mechanically will see the associated Hawking radiation [29] since this phenomenon is only a consequence of doing matter field theory in the background geometry. They will infer a finite entropy and wonder how to microscopically account for it.

## 9 Cosmologies

If  $\Lambda$  is truly a constant then the  $dS_4$  solution describes a state of permanent inflation. However, what is usually meant by inflation occurs when a metastable matter vacuum dominates the energy density, but ends when this state relaxes in some way to a true vacuum with negligible vacuum energy. Clearly this is impossible in the dilaton theory because inflation is caused by negative energy density which cannot relax to zero.

Let us seek other homogeneous and isotropic cosmological solutions in (non-relativistic) matter-dominated and radiation-dominated regimes, and compare them to standard (spatially flat say) Friedman-Robertson-Walker cosmologies. Clearly  $\phi$  will be a function of time only. To facilitate the comparison note that since matter and radiation couple to the dilaton via their generally covariant couplings to the auxiliary metric, they cannot distinguish this metric from a non-conformal time-coordinate transformation of the metric,

$$ds^2 = d\tau^2 - a^2(\tau)d\vec{x}^2, \quad (9.1)$$

where  $d\tau/dt = \phi(t)$ ,  $a(\tau) = \phi(t)$ . In a matter-dominated era we have the usual scaling of the energy density,  $\rho(\tau).a^3(\tau) = \rho_0 = \text{constant}$ . That is,  $\rho_0$  is the fixed mass density of matter with

respect to *coordinate-volume*. This allows us to straightforwardly generalize the point source of Section 5 to this density using superposition, which imposing homogeneity yields,

$$\partial_t^2 \phi = -\frac{\rho_0}{M^2}. \quad (9.2)$$

The simple solution is

$$\phi(t) = -\frac{\rho_0}{2M^2}t^2 + \phi_1 t + \phi_0, \quad (9.3)$$

where  $\phi_{0,1}$  are integration constants. By a shift of the origin of the time coordinate we can put this in the simpler form,

$$\phi(t) = \phi_0 - \frac{\rho_0}{2M^2}t^2. \quad (9.4)$$

In standard gravity, matter domination gives  $a \propto \tau^{2/3}$ , which is easily seen to correspond to  $\phi \propto t^2$ . This appears similar to the scalar gravity result but there is a crucial sign difference in the  $t^2$ -dependence, once again traced to the fact that the dilaton has canonical-sign kinetic term while the conformal mode of standard gravity does not. In the scalar gravity case we see that the physical regime,  $\phi > 0$ , gives the universe a finite lifetime between a Big Bang and a Big Crunch, where we have  $\phi = 0$  and can therefore not trust the chiral Lagrangian predictions. But these cosmological singularities must be somehow resolved by the fundamental conformal field theory dynamics.

Let us now consider radiation dominance. This case is even easier, because as we have earlier noted, radiation does not couple to the dilaton. Therefore we have simply,

$$\partial_t^2 \phi = 0, \quad (9.5)$$

with solution

$$\phi = \phi_1 t + \phi_0. \quad (9.6)$$

If  $\phi_1 \neq 0$  then we can remove  $\phi_0$  by a shift of the time coordinate. This clearly yields

$$a(\tau) = \sqrt{2\phi_1} \tau^{1/2}, \quad (9.7)$$

which is very similar to the standard radiation-dominated FRW cosmology, but the Hubble parameter is not related to the radiation energy density. There is another possible solution when  $\phi_1 = 0$ , which yields

$$a(\tau) = \phi_0, \quad (9.8)$$

which corresponds to Minkowski space, despite the presence of radiation.

As will be discussed in Section 15, there is a higher-dimensional embedding of the four-dimensional dilaton effective theory, in which form the above cosmological solutions were first studied [30]. In particular, it was pointed out that the crucial sign differences we have seen occurring in the scalar and standard cosmologies implies a very different unfolding of the universe and condensation of its elements.



## 10 The Cosmological Constant Problem

As discussed in Section 2, taking  $\Lambda \ll M^4$  as we must to have a sensible effective field theory of the broken phase of a CFT, appears unnatural. With the inclusion of matter we see that we are dealing with an almost exact analog of the usual cosmological constant problem. Matter couples to the dilaton via the auxiliary metric  $g_{\mu\nu} \equiv \phi^2 \eta_{\mu\nu}$  in exactly the usual generally covariant fashion, so the matter vacuum energy, classical plus quantum corrections, contributes to our cosmological constant  $\Lambda$  exactly as it would in the case of the same matter coupled to real gravity. An important qualification is that the equality of matter vacuum energy contributions to the dilatonic and standard gravity cosmological constants is only guaranteed if one uses the same UV regularization, for example dimensional regularization. Not only are the contributions the same, the effects of these constants in obstructing Poincare invariant solutions to the equations of motion is also very similar, as discussed in the previous section. Of course, the subleading (Planck-suppressed) quantum gravity and quantum dilaton corrections to the matter vacuum energy are certainly different in detail, but this is not the most robust aspect of the cosmological constant problem.

I find this close analogy of the technical face of the cosmological constant problem very tantalizing. In a sense the analog of gravity is now much simpler, just a scalar field. Yet the cosmological constant problem seems essentially the same, and just as hard. Is it the case nevertheless that other differences between real gravity and the dilaton are essential for solving the problem in the former case, or can both the real and analog problems be solved by the same basic mechanism?

Ref. [31] derived a useful No-Go theorem to filter out a large class of proposals for dynamical adjustment of the cosmological constant by light matter fields. The derivation makes central use of the trace of Einstein's equations. This is precisely the Einstein equation which follows by varying metrics of the special form  $g_{\mu\nu} = \phi^2(x) \eta_{\mu\nu}$ . Further, the derivation is insensitive to the sign flip discussed in Section 2 between the dilatonic and standard Einstein kinetic terms. Therefore the derivation and no-go theorem apply to the dilatonic theory.

## 11 Ultraviolet Completion

Our non-renormalizable effective theory of the dilaton coupled to other light remnants of spontaneous conformal invariance breaking must be UV-completed by a conformal field theory (CFT) with a continuum limit. To have a broken phase it must possess a (at least approximate) moduli space of degenerate vacua parametrized by the VEV of the corresponding Goldstone boson,  $\phi$ . The moduli space may have several such directions breaking conformal invariance. At the

origin of moduli space the conformal invariance is intact, but anywhere else it is spontaneously broken.

Such a CFT would be to scalar gravity what string theory is for real gravity, the UV completion of the non-renormalizable effective theory. Just as with string theory, the simplest constructions come with supersymmetry, for example  $N = 4$  super-Yang-Mills theory or the conformal field theories arising in  $N = 1$  SQCD in the conformal window. It is much more difficult to find interacting, fundamentally non-supersymmetric CFT's with (approximate) moduli spaces. Just as in the case of string theory and real gravity, this does not prove that such non-supersymmetric theories do not exist, but certainly the supersymmetric examples are easier to find.

At first sight it may seem like a simplification that the UV completion of the scalar gravity theory can still be a field theory, just a CFT, while quantum gravity requires going outside field theory to string theory. However, the distinction has somewhat diminished with the advent of the AdS/CFT correspondence [10]. See Sections 15 and 16.

## 12 Sub-Planckian length scales?

In ordinary quantum gravity it is difficult to attach meaning to distances smaller than the Planck length. In the dilaton theory, at distances smaller than  $1/M$  the non-renormalizable chiral lagrangian description certainly breaks down, but the fundamental CFT description is still valid and there is a Minkowski spacetime in which CFT matter propagates. But even though the theory is under control, the restored conformal invariance at distances below  $1/M$  certainly makes distance a less meaningful experimental quantity, whereas at larger distances the concept is as useful as in the real world. Earlier studies of the distance limit in real gravity [32], but restricted to the quantum dynamics of the conformal factor for simplicity, naturally relate to the case of the dilaton theory.

## 13 Dilaton Mass

Recently it has been shown [11] that one can make effective field theory sense of quantum General Relativity weakly deformed by graviton interactions that violate general coordinate invariance. In particular, the graviton can be given a small mass. One price for this violation of the “gauge symmetry” is that the cutoff imposed by non-renormalizability on the effective theory is lowered below the Planck scale to a weighted geometrical mean of the Planck scale and the graviton mass. A second price is that although string theory gives a good account of

what might UV complete ordinary General Relativity, there is no known candidate for such a completion in the deformed case.

The analogous issues in the dilaton case are clearer and more satisfying, and perhaps may shed light on how things might ultimately work in real gravity. The basic plot is that the fundamental UV theory is not a CFT but rather an asymptotically free theory which flows to an infrared attractive CFT. This CFT contains a moduli space as discussed in the last section. We imagine living away from the origin of moduli space in a field direction which we call  $\phi$ , the dilaton. An explicit example of such a situation is given by SQCD theories in the conformal window. Thus, the protective symmetry of our dilaton effective field theory, namely (spontaneously broken) conformal invariance, is not an exact symmetry of the dynamics, but rather an accidental IR symmetry. If one considered the vacuum at the origin of the moduli space then the RG flow of the fundamental UV theory would get arbitrarily close to the attractive fixed point associated with the CFT. However, with  $\langle\phi\rangle \neq 0$ , this RG flow is interrupted at  $\langle\phi\rangle M$ , so that there is some residual deviations from exact conformal invariance of the dynamics. These residual deviations can lead to a stabilized dilaton with non-zero mass [33] [34].

In more detail [34], suppose the fundamental UV theory gets close enough to the infrared fixed point once we have run down to a scale  $\Lambda_{CFT}$  that we can begin trusting the RG flow of the CFT linearized about the associated fixed point. We can write the effective lagrangian at this scale as

$$\mathcal{L}(\Lambda_{CFT}) = \mathcal{L}_{fixed-point} + \sum_n g_n(\Lambda_{CFT}) \mathcal{O}_n(x), \quad (13.1)$$

where the  $\mathcal{O}_n(x)$  are a basis of scaling operators for the CFT, which are irrelevant since the CFT is IR-attractive. The dimensionless coefficients  $g_n(\Lambda_{CFT}) \sim \mathcal{O}(1)$  (so that we are at the border of being able to linearize the RG in these couplings). This linearized flow is given by

$$\mu \frac{d}{d\mu} g_n(\mu) = \gamma_n g_n(\mu), \quad (13.2)$$

where  $\gamma_n > 0$  are the anomalous dimensions (deviations of the scaling dimensions from four) of the associated operators. They govern the running down to the scale  $\langle\phi\rangle M$  where the effective chiral lagrangian for the dilaton and other light remnants takes over. Running down to  $\langle\phi\rangle M \ll \Lambda_{CFT}$  yields

$$\begin{aligned} \mathcal{L}(\langle\phi\rangle M) &= \mathcal{L}_{fixed-point} + \sum_n g_n(\langle\phi\rangle M) \mathcal{O}_n(x) \\ &= \mathcal{L}_{fixed-point} + \sum_n \left( \frac{\langle\phi\rangle M}{\Lambda_{CFT}} \right)^{\gamma_n} g_n(\Lambda_{CFT}) \mathcal{O}_n(x). \end{aligned} \quad (13.3)$$

To go below the  $\langle\phi\rangle M$  threshold we must match onto the dilatonic effective theory but now including the small perturbations with different scaling properties from the exact CFT. For simplicity let us consider only the most relevant of the operators  $\mathcal{O}_n$ , calling it simply  $\hat{\mathcal{O}}$ , with coupling  $g$  and anomalous dimension  $\gamma > 0$ . This perturbation can and generically will match onto a small correction to the dilatonic effective theory, the most relevant such effect being on the dilaton potential,

$$\mathcal{L}_{\text{dilaton}} = \frac{M^2}{2}(\partial\phi)^2 - \Lambda\phi^4 + \mathcal{O}(1)\left(\frac{\phi M}{\Lambda_{\text{CFT}}}\right)^\gamma g(\Lambda_{\text{CFT}})M^4\phi^4 + \text{less relevant}, \quad (13.4)$$

where we have used the fact that  $M\phi$  is the only scale (up to derivatives) that can saturate the canonical dimension (four) of  $\hat{\mathcal{O}}$  in the matching to the effective Lagrangian. It is straightforward to see that the resulting potential violates conformal invariance and can stabilize the dilaton in the broken phase and generate a small mass for it [34] (if  $\Lambda$  is small as we have assumed all along).

## 14 Supersymmetry (Breaking)

Making our chiral lagrangian exactly supersymmetric is straightforward enough, by elevating the dilaton to a chiral superfield. Similarly matter comes in supermultiplets. So for example, the pure dilatonic theory would be a massless Wess-Zumino theory now, the cosmological constant arising from the superpotential. The dilaton superfield now couples to matter in a manner almost identical to the auxiliary conformal compensator of supergravity, except that the dilaton multiplet has the right-sign kinetic term since it has propagating degrees of freedom, the dilaton, “axion” and dilatino. As a consequence, supersymmetric anomaly-mediation [14] is very similar in supergravity and the dilaton theory. Clearly, unbroken supersymmetry can protect the analog cosmological constant just as in supergravity theories.

Our central problem occurs if we imagine that the matter sector is not supersymmetric or at least has broken supersymmetry, just as is the case of the Standard Model in the real world. Such supersymmetry breaking might arise in one of three ways.

(a) The UV completion is some (as yet unknown) non-supersymmetric CFT with an (approximate) modulus. Then there is no mystery as to why its infrared remnants are non-supersymmetric. This is the analog of talking about fundamentally non-supersymmetric strings.

(b) The CFT is exactly supersymmetric but, as in the previous section, the fundamental theory is not this SCFT but rather an asymptotically free theory that flows in the infrared to the SCFT. Now suppose that this fundamental theory is NOT supersymmetric, that supersymmetry is just an accidental symmetry of the CFT. However, as discussed in the previous section the

RG flow never reaches the fixed point of the CFT because the running stops at  $\langle\phi\rangle M$ , and therefore the accidental supersymmetry does not become exact. Rather it appears in the effective dilaton theory as a form of weak but explicit supersymmetric breaking, accompanying the weak breaking of the conformal symmetry of the dynamics [35].

(c) The CFT is the UV completion and is supersymmetric but the vacuum we expand about is not supersymmetric, that is supersymmetry is spontaneously broken. What I find intriguing about this case is that it is possible to prove [12] the analog of a conjecture that has been made in superstring theory [13], namely that there are no Poincare invariant vacua which are not supersymmetric. That is, broken supersymmetry must be accompanied by a cosmological constant which obstructs Poincare invariant solutions.

In the dilaton case a simple proof can be given [12]. Since the fundamental SCFT has global super-Poincare-invariant dynamics (ungauged by supergravity) the order parameter for supersymmetry breaking is simply the (positive) vacuum energy density,  $\Lambda$ . If this does not vanish then it also spontaneously breaks conformal invariance. In order to have exact (but non-linearly realized) conformal invariance of the dilaton effective lagrangian, this vacuum energy must be dressed with the dilaton field to become our analog cosmological constant,

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 - \Lambda\phi^4. \quad (14.1)$$

With such a non-zero cosmological constant a Poincare invariant vacuum becomes impossible. Of course we can also choose the “non-geometric” phase at the origin of moduli space  $\langle\phi\rangle = 0$  where both supersymmetry and conformal invariance (including Poincare invariance) are preserved.

## 15 Localized Dilaton in Higher-dimensional Gravity

The Randall-Sundrum II (RS2) braneworld scenario [15] is well known to demonstrate how a massless 4D graviton mode (and attendant 4D General Relativity) localized about a 3-brane can emerge from a higher-dimensional gravity theory, where matter fields are taken to be confined to the 3-brane. Macroscopically this leads to 4D gravity coupled to 4D matter. What is less well known is that there is an exact analog of this where the 4D graviton mode is replaced by a 4D dilaton, so that macroscopically we recover our effective dilaton theory.

The localized dilaton model (see Refs. [33] [34] for this interpretation) is reached by beginning with the original RS1 model [36] (without stabilizing the radius). The 4D effective field theory below the masses of KK excitations [37] [38] involves the light matter fields on the “IR” brane as well as the massless 4D graviton and the massless 4D radion,  $r_c(x)$ , whose VEV is the

extra-dimensional radius,

$$\mathcal{L}_{4Def} = \sqrt{-g} \left\{ \frac{1 - e^{-2k\pi r_c(x)}}{k} M_5^3 R + \frac{M_5^3}{k} g^{\mu\nu} \partial_\mu e^{-k\pi r_c(x)} \partial_\nu e^{-k\pi r_c(x)} \right\} + \sqrt{-g_{IR}} \mathcal{L}_{IRmatter}(g_{IR}), \quad (15.1)$$

where  $g_{IR\mu\nu} = g_{\mu\nu}(x)e^{-2k\pi r_c(x)}$  is the induced metric on the IR brane and  $\mathcal{L}_{IRmatter}(g_{IR})$  is a general lagrangian for matter localized on the IR brane and covariantly coupled to the induced metric. Making the field redefinition

$$\phi \equiv \frac{e^{-k\pi r_c(x)}}{e^{-k\pi \langle r_c \rangle}} \quad (15.2)$$

we get,

$$\mathcal{L}_{4Def} = \sqrt{-g} \left\{ \frac{1 - \phi^2 e^{-2k\pi \langle r_c \rangle}}{k} M_5^3 R + \frac{M_5^3 e^{-2k\pi \langle r_c \rangle}}{k} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} + \sqrt{-g_{IR}} \mathcal{L}_{IRmatter}(g_{IR}), \quad (15.3)$$

and

$$g_{IR\mu\nu} = g_{\mu\nu}(x) \phi^2. \quad (15.4)$$

Now clearly the effective 4D Planck scale of the 4D graviton is approximately  $\sqrt{M_5^3/k}$ . We can therefore decouple this dynamical 4D gravity by taking the limit  $M_5^3/k \rightarrow \infty$ . We will choose to do it holding  $M^2 \equiv 2e^{-2k\pi \langle r_c \rangle} M_5^3/k$  fixed, and expanding about  $g_{\mu\nu} = \eta_{\mu\nu}$ . Since we have decoupled gravitational fluctuations we have exactly  $g_{\mu\nu} = \eta_{\mu\nu}$ . In this limit then,

$$\mathcal{L}_{4Def} = M^2 (\partial\phi)^2 + \sqrt{-g_{IR}} \mathcal{L}_{IRmatter}(g_{IR}), \quad (15.5)$$

and

$$g_{IR\mu\nu} = \eta_{\mu\nu} \phi^2. \quad (15.6)$$

This is precisely the form of our dilatonic effective field theory. While in phenomenological RS1 applications one usually chooses the scale we defined as  $M$  to be of order the weak scale, in our dilatonic analog we choose it to be the “Planck” scale. The fine-tuning of the analog cosmological constant is reflected in the tuning of the IR brane tension in RS1, while departures from this tuned case give rise to “bent brane” behaviors [39] [40] that reflect the  $AdS_4$  and  $dS_4$  behaviors we saw earlier. The more general cosmologies we found before correspond to the RS1 cosmologies studied in Ref. [30] (without radius stabilization).

The limit of RS1 we took, whose 4D effective theory is the dilaton effective theory, has a simple spacetime interpretation. We are keeping the IR brane fixed and sending the Planck brane infinitely far away. (It is the opposite of the move we make to get from RS1 to RS2, where we keep the Planck brane fixed and send the IR brane infinitely far away.) Remarkably the radion mode is localized on the IR brane and therefore survives this limit and is identified with

the dilaton. The localized dilaton model is this limiting case of RS1. The full 5D model also contains massive KK gravitons, which of course decouple for the most part in the low energy theory. In this way there are black holes associated to the higher-dimensional completion of the dilatonic theory, but they are 5D black holes. Such black holes may (I do not know a proof) also encompass the IR brane [41], seeded by concentrations of brane matter, thereby answering the issue raised in Section 5 of what happens when matter lies within its “Schwarzschild radius”,  $r_S$ . If this is true then the difference with ordinary gravity is in the dimensionality of the black hole that forms! Even light, which we saw in Section 6 is usually hard to bend, could then be trapped within a 5D black hole encompassing the IR brane. In any case in this RS picture it is the KK graviton modes of the UV completion of the dilaton effective theory (in totality a string theory on the RS background) that resolves the singularity at  $r_S$  (for large  $r_S$ ) found in Section 5.

## 16 AdS/CFT

We normally approach the  $\text{AdS} \equiv \text{CFT}$  correspondence [10] starting from the left-hand side, imagining string theory as a UV completion of quantum gravity in such a background. It is then a surprise that this is dual to doing CFT without gravity. The direction is reversed in our dilatonic analogy. We imagine doing CFT as the UV completion of our dilatonic effective field theory. To our surprise (at least if the CFT has a large gap in the spectrum of scaling dimensions and a large- $N$  type expansion) we find that the CFT dynamics is dual to a true gravitational theory in  $\text{AdS}_5$ . Since we imagine the CFT to be spontaneously broken, the full conformal symmetry of  $\text{AdS}_5$  is not realized, the space is truncated before the horizon. In warped effective field theory this is captured as in the previous section [42] [33] [34].

## 17 Euclidean Continuation

Technically and conceptually it is often useful to have a Euclidean continuation of quantum field theory in Minkowski space. The path integrals associated with the Euclidean continuations are usually better defined, allowing some understanding of non-perturbative effects. In the case of quantum General Relativity the Euclidean path integral has a notoriously ill-defined measure for Riemannian metrics, stemming from the unboundedness of the Euclidean Einstein action [16]. This is due to the “wrong-sign” kinetic term for the conformal mode of the dynamical metric. There is no such problem for the dilaton theory coupled to light matter because the dilaton has a canonical kinetic term.

In Euclidean quantum gravity one expects to sum over different topologies. In the dilatonic case, especially in the generally covariant formulation given at the end of Section 2, we see that different topologies can be related to flat space by globally non-trivial conformal transformations. It would be interesting to study whether these non-trivial topologies must be included in the dilatonic effective Euclidean path integral in order to match the globally non-trivial IR consequences of the fundamental CFT.

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